The Dividend Discount Model

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In this white paper we will derive the Dividend Discount Model equation. For that purpose we will use the following hypothetical problem...

Our Hypothetical Problem

Assume that we are tasked with calculating the enterprise value of ABC Company given the following model parameters...

Table 1: Model Parameters

Description	Value
Free cash flow in month zero	1,0000
Annualized growth rate of free cash flow	5.00%
Annualized cost of capital	15.00%

Question: What is the enterprise value of ABC Company?

Building Our Model

We will define the variable C_m to be free cash flow in month m. Using Table 1 above the equation for free cash flow in month zero is...

$$C_0 = 1,000.00 \tag{1}$$

We will define the variable g to be the monthly growth rate of free cash flow. Using Table 1 above the equation for the monthly free cash flow growth rate is...

$$g = \Bigg(1 + \text{annualized growth rate}\Bigg)^{\frac{1}{12}} - 1 = 1.05^{\frac{1}{12}} - 1 = 0.00407 \tag{2}$$

We will define the variable k to be the monthly cost of capital. Using Table 1 above the equation for the monthly cost of capital is...

$$k = \Bigg(1 + \text{annualized cost of capital}\Bigg)^{\frac{1}{12}} - 1 = 1.15^{\frac{1}{12}} - 1 = 0.01171 \tag{3}$$

We will define the variable V_0 to be the present value of free cash flow at time zero (i.e. enterprise value). Using Equations (1), (2) and (3) above the equation for enterprise value is...

$$V_0 = \sum_{m=1}^{\infty} C_0 \left(1+g\right)^m \left(1+k\right)^{-m} = \sum_{m=1}^{\infty} C_0 \Bigg(\frac{1+g}{1+k}\Bigg)^m \tag{4}$$

Note that we can rewrite Equation (4) above as...

$$V_0 = C_0 \sum_{m=1}^{\infty} \theta^m \ \text{ ...where... } \ \theta = \frac{1+g}{1+k} \tag{5}$$

Note that the solution to a polylogarithm of order zero is... [1]

$$Li_0(z) = \sum_{k=1}^{\infty} z^k = \frac{z}{1-z} \ \text{ ...where... } \ |z| < 1 \tag{6}$$

Using Equation (6) above the solution to summation in Equation (5) above is...

$$\sum_{m=1}^{\infty} \theta^m = \frac{\theta}{1-\theta} = \frac{\frac{1+g}{1+k}}{1-\frac{1+g}{1+k}} = \frac{\frac{1+g}{1+k}}{\frac{1+k-1-g}{1+k}} = \frac{1+g}{k-g} \quad \text{...given that...} \quad g < k \tag{7}$$

Using Equation (7) above the solution to Equation (5) above is...

$$V_0 = \frac{C_0\,(1+g)}{k-g} \tag{8}$$

Note that Equation (8) above is the equation for enterprise value given free cash flow in month zero. Since free cash flow grows at the monthly rate g we can rewrite that equation as...

$$V_0 = \frac{C_1}{k-g} \quad \text{...given that...} \quad C_1 = C_0\,(1+g) \tag{9}$$

The Answer To Our Hypothetical Problem

Using the equations above the answer to our problem is...

$$V_0 = \frac{C_0\,(1+g)}{k-g} = \frac{1,000 \times (1+0.00407)}{0.01171 - 0.00407} = 131,400 \tag{10}$$

References

[1] Gary Schurman, *Polylogarithm Of Order Zero,* May, 2019